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Research Article

On Decay of Dilatational Acceleration Waves in Thermoelasticity

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Abstract

Our study deals with a thermoelastic micropolar material with voids including the time derivative among the independent constitutive variables.

Within this theory we formulate the mixed initial-boundary value problem and prove an uniqueness result regarding the solution of this problem and analyze the decay of dilatational acceleration waves.

Introduction

Materials which operate at elevated temperature will invariable be subjected to heat flow at some time during normal use. The heat flow will involve a temperature distribution which will inevitably give rise to thermal stresses.

The role of the pertinent material properties and other variables can affect the magnitude of thermal stress must be well understood and all possible mode of failure must be considered.

Our following considerations can be useful in other fields of applications which deal with porous materials as geological materials, solid packed granular and many others. Beginning of the investigations on porous materials was the Goodman and Cowin theory of granular theory [1]. In this theory and also in the theory of Cowin and Nunziato [2], the authors introduce an additional degree of freedom in order to develop the mechanical behavior of porous solids in which the matrix material is elastic and the interstices are voids material. interstices are voids of material. The intended applications of this theory are to geological materials like rocks and soil and to manufactured porous materials, like ceramics and pressed powders. The theory of Cowin and Nunziato (see also [3]) is dedicated to non conductor of heat materials. The basic premise underling this theory is the concept of a material for which the bulk density is written as the product of two fields, the matrix material density field and the volume fraction field (see also, [4-6]). The theory of thermoelastic materials with voids proposed by Iesan in [7] is a straightforward generalization of the linear elastic body, but the author neglected that the changes in the volume fraction result in internal dissipation in the material. Other results regarding the bodies with microstructure can be find in studies [8-13].

In our study we extend the Cowin and Nunziato theory to cover the micropolar material by adding into the set of constitutive variables the time derivative of the voidage to include the inelastic effects.

Basic Equations

Consider a body that at time $t = 0$ occupies a properly regular region B of Euclidean three-dimensional space R^3. Assume that the boundary of B, denoted by ∂B, is a sufficiently smooth surface to admit the application of divergence theorem. Also, we denote the closure of B by B^-. Throughout this paper we refer the motion of the continuum to a fixed system of rectangular Cartesian axes Ox_i, (i = 1, 2, 3) and adopt Cartesian tensor notation. The italic indices will always assume the values 1, 2, 3, whereas Geek indices will range over the value 1,2. A superposed dot stands for the material time derivative while a comma followed by a subscript denotes partial derivatives with respect to the spatial respective coordinates. Einstein summation on repeated indices is also used. Also, the spatial argument and the time argument of a function will be omitted when there is no likelihood of confusion.

In the reference configuration the bulk density q, the matrix density γ and the matrix volume fraction ν are related by

$$\rho_o = \gamma_o \nu_o$$

where γ_0 and v_0 are spatially constants. The independent variables which describe the motion of the micropolar thermoelastic body with voids are:

- $u_i(x, t)$, $\varphi_i(x, t)$ - the displacement and microrotation fields from reference config- uration;
- θ - the change in temperature from T_0, the absolute temperature of the reference configuration, i.e. $\theta(x, t) = T(x, t) T_0$;
- σ - the change in volume fraction measured from the reference configuration volume fraction vo, i.e. $\sigma(x, t) = v(x, t) v_0$.

Supposing that the initial body is stress free, with zero intrinsic equilibrated body force and zero flux rate, we can write the free energy function, within linear theory, as follows:

$$\varrho\Psi = \begin{array}{l} \frac{1}{2}A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + B_{ijmn}\varepsilon_{ij}\gamma_{ij} + \frac{1}{2}C_{ijmn}\gamma_{ij}\gamma_{mn} + \\ +B_{ij}\sigma\varepsilon_{ij} + C_{ij}\sigma\gamma_{ij} + D_{ijk}\sigma_{,k}\varepsilon_{ij} + E_{ijk}\sigma_{,k}\gamma_{ij} - \\ -\beta_{ij}\theta\varepsilon_{ij} - \alpha_{ij}\theta\gamma_{ij} - m\theta\sigma + d_i\sigma\sigma_{,i} + a_i\theta\sigma_{,i} - \\ -\frac{1}{2}a\theta^2 + \frac{1}{2}\xi\sigma^2 + \frac{1}{2}A_{ij}\sigma_{,i}\sigma_{,j} - \frac{1}{2}\omega\dot{\sigma}^2. \end{array} \text{ ----- (1)}$$

As in [2], $f = -\omega\sigma^{\cdot}$ is the dissipation which takes into account of the inelastic behavior of the voids. Also, ω is a positive constant. By using an usual procedure, with the aid of the free energy function, we can derive the following constitutive equations:

$$\begin{array}{l} t_{ij} = C_{ijmn}\varepsilon_{mn} + B_{ijmn}\gamma_{mn} + B_{ij}\sigma + D_{ijk}\sigma_{,k} - \beta_{ij}\theta, m_{ij} \\ = B_{mnij}\varepsilon_{mn} + C_{ijmn}\gamma_{mn} + C_{ij}\sigma + E_{ijk}\sigma_{,k} - \alpha_{ij}\theta, \\ h_i = A_{ij}\sigma_{,j} + D_{mni}\varepsilon_{mn} + E_{mni}\gamma_{mn} + d_i\sigma - a_i\theta, \\ g = -B_{ij}\varepsilon_{ij} - C_{ij}\gamma_{ij} - \xi\sigma - d_i\sigma_{,i} + m\theta,\ \eta = \\ \beta_{ij}\varepsilon_{ij} + \alpha_{ij}\gamma_{ij} + m\sigma + a_i\sigma_{,i} + a\theta, \\ qi = kij\theta, j, \end{array} \text{ ----- (2)}$$

where ε_{ij} and γ_{ij} are the kinematic characteristics of the strain and we have the following geometric relations:

$$\varepsilon_{ij} = u_{j,i} + \varepsilon_{jik}\phi_k, \gamma_{ij} = \phi_{j,i}, \theta = T - T_0, \sigma = \nu - \nu_0. \text{ ---- (3)}$$

With the aid of a procedure similar to that used by Nunziato and Cowin in [3], we obtain the following fundamental equations (see also, [9, 10]):

- the equations of motion:

$$\begin{array}{l} t_{ij,j} + \rho F_i = q\ddot{u}_i, \\ m_{ij,j} + \varepsilon_{ijk}t_{jk} + qM_i = I_{ij}\ddot{\varphi}_j; \end{array} \text{ ----- (4)}$$

- the balance of the equilibrated forces:

$$h_{i,i} + g + \rho L = \rho\kappa\ddot{\sigma}; \text{ ----- (5)}$$

- the energy equation:

$$\rho T_0\dot{\eta} = q_{ij} + \rho r. \text{ ----- (6)}$$

In the above equations we have used the following notations:

q - the constant mass density;
η - the specific entropy;
T_o - the constant absolute temperature of the body in its reference state;
I_{ij} - coefficients of microinertia;
κ - the equilibrated inertia;
u_i - the components of displacement vector;
φ_i - the components of microrotation vector;
φ - the volume distribution function which in the reference state is φ_o;
σ - the change in volume fraction measured from the reference state;
θ - the temperature variation measured from the reference temperature T_0; ε_{ij}, γ_{ij}-kinematic characteristics of the strain;
t_{ij} - the components of the stress tensor;
m_{ij} - the components of the couple stress tensor;
h_i - the components of the equlibrated stress vector;
q_i - the components of the heat flux vector;
F_i - the components of the body forces;
M_i - the components of the body couple;
r - the heat supply per unit time;
g - the intrinsic equilibrated force;
L - the extrinsic equilibrated body force;

A_{ijmn}; B_{ijmn}; ....., k_{ij} - the characteristic functions of the material, and they obey the symmetry relations

$$A_{ijmn} = A_{mnij}, C_{ijmn} = C_{mnij}, k_{ij} = k_{ji}. \text{ ----- (7)}$$

The entropy inequality implies

$$-\frac{1}{T_0}k_{ij}\theta_{,i}\theta_{,j} - \omega\dot{\sigma}^2 \leq 0. \text{ ----- (8)}$$

The equations (4) and (6) are analogous to the classical equations of motion and, re- spectively, to the balance equation, whereas the new balance of equilibrated force (5) can be motivated by a variational argument as in [14].

The mixed initial-boundary value problem within context of thermoelastic theory of micropolar bodies with voids is complete if we give the boundary and initial conditions. So, the boundary conditions can be deduced as in [15] and ve must give the additional data for the surface continuous temperature field on the boundary ∂B of the geometry of the body B and for the time interval for which the solution is desired. Also, we must add the initial temperature field. In conclusion, we have the following initial conditions:

$$\begin{array}{l} u_i(x,0) = u_i^0(x), \dot{u}_i(x,0) = u_i^1(x), x \in \bar{B}, \\ \varphi_i(x,0) = \varphi_i^0(x), \dot{\varphi}_i(x,0) = \varphi_i^1(x), x \in \bar{B}, \\ \theta(x,\theta) = \theta^0(x), \sigma(x,0) = \sigma^0(x), \dot{\sigma}(x,0) = \sigma^1(x), x \in \bar{B}, \end{array} \text{ ----- (9)}$$

and the following prescribed boundary conditions

$$\begin{array}{l} u_i = \bar{u}_i \text{ on } \partial B_1 \times [0,t_0], t_i \equiv t_{ij}n_j = \bar{t}_i \text{ on } \partial B_1^C \times [0,t_0], \\ \varphi_i = \overline{\varphi}_i \text{ on } \partial B_2 \times [0,t_0],\ m_i \equiv m_{ij}n_j = \overline{m}_i \text{ on } \partial B_2^c \times [0,t_0], \\ \sigma = \overline{\sigma} \text{ on } \partial B_3 \times [0,t_0],\ h \equiv h_i n_i = \overline{h} \text{ on } \partial B_3^c \times [0,t_0], \\ \theta = \overline{\theta} \text{ on } \partial B_4 \times [0,t_0],\ q \equiv q_i n_i = \overline{q} \text{ on } \partial B_4^c \times [0,t_0], \end{array} \text{ --(10)}$$

where ∂B_1, ∂B_2, ∂B_3 *and* ∂B_4 with respective complements ∂B^c_1,

∂B^c_2, ∂B^c_3 *and* ∂B^c_4 are subsets of ∂B, n_i are the components of the unit outward normal to ∂B, t_0 is some instant that may be infinite, $u_i^0, u_i^1, \varphi_i^0, \varphi_i^1, \theta^0, \sigma^0, \sigma^1, \overline{u}_i, \overline{t}_i, \overline{\varphi}_i, \overline{m}_i, \overline{\sigma}, \overline{\theta}, \overline{q}$ and h^- *are prescribed functions in their domains.*

By a solution of the mixed initial-boundary value problem for the thermoelasticity of micropolar bodies with voids, in the cylinder $\Omega_0 = B \times [0, t_0]$ we mean an ordered array (u_i, φ_i, σ, θ) which satises the equations (4)-(6) for all (x; t) ε Ω_0, the boundary conditions (9) and the initial conditions (10). Introducing equations (2) and (3) into equations (4), (5) and (6), we obtain the following system of equations.

$$\begin{aligned}
&\rho \ddot{u}_i = (A_{ijmn}\varepsilon_{mn} + B_{ijmn}\gamma_{mn} + B_{ij}\sigma + D_{ijk}\sigma_{,k} - \beta_{ij}\theta)_{,j} + \rho F_i,\\
&I_{ij}\ddot{\varphi}_j = (B_{mnij}\varepsilon_{mn} + C_{ijmn}\gamma_{mn} + C_{ij}\sigma + E_{ijk})\sigma_{,k} - \alpha_{ij}\theta)_{,j} +\\
&\varepsilon_{ijk}(A_{jkmn}\varepsilon_{mn} + B_{jkmn}\gamma_{mn} + B_{jk}\sigma + D_{jkm}\sigma_{,m} - \beta_{jk}\theta) + \rho M_i, \qquad \text{---(11)}\\
&\rho\kappa\ddot{\sigma} = (D_{mni}\varepsilon_{mn} + E_{mni}\gamma_{mn} + d_i\sigma + A_{ij})\sigma_{,j} - a_i\theta)_{,j} + \rho L -\\
&B_{ij}\varepsilon_{ij} - C_{ij}\gamma_{ij} - \xi\sigma - d_i\sigma_{,i} + m\theta,\\
&a\dot{\theta} = \frac{1}{\rho T_0}(k_{ij}\theta_{,j})_{,i} + \frac{1}{T_0}r - \beta_{ij}\dot{\varepsilon}_{ij} - \alpha_{ij}\dot{\gamma}_{ij} - m\dot{\sigma} - a_i\dot{\sigma}_{,i}.
\end{aligned}$$

Main Results

In the first part of this section, we prove the uniqueness of solution for the mixed problem within context of above thermoelastodynamics by using an energetic method.

Consider two solutions ($u_i^{(1)}$, $\varphi_i^{(1)}$, $\sigma^{(1)}$, $\theta^{(1)}$) and ($u_i^{(2)}$, $\varphi_i^{(2)}$, $\sigma^{(2)}$, $\theta^{(2)}$) of our problem for the same body B, subjected to the same body force F_i, the same body couple M_i, the same extrinsic equilibrated body force L and the same heat supply r. For each solution we have an appropriate set of boundary and initial conditions of the same kind of (10) and (9). Because of the linearity of the eld equations and conditions, the difference of two solutions is also a solution of our problem, but corresponding to the null body force, body couple, heat supply, extrinsic equilibrated body force and null boundary and initial data.

Let $(\overline{u}_i, \overline{\varphi}_i, \overline{\sigma}, \overline{\theta})$ be the difference of two solutions, that is

$$\overline{u}_i = u_i^{(2)} - u_i^{(1)}, \overline{\varphi} = \varphi_i^{(2)} - \varphi_i^{(1)}, \overline{\sigma} = \sigma^{(2)} - \sigma^{(1)}, \overline{\theta} = \theta^{(2)} - \theta^{(1)}.$$

We also denote with superposed bar the quantities corresponding to the difference of two solutions, for instance $\overline{t}_{ij} = t_{ij}^{(2)} - t_{ij}^{(1)}$.

The dierential equations governing the difference solutions are:

$$\overline{t}_{ij,j} = \rho\ddot{\overline{u}}_i,$$

$$\overline{m}_{ij,j} + \varepsilon_{ijk}\overline{t}_{jk} = I_{ij}\ddot{\overline{\varphi}}_j; \qquad \text{----- (12)}$$

$$\overline{h}_{i,i} + \overline{g}\kappa\ddot{\overline{\sigma}}; \qquad \text{----- (13)}$$

$$\rho T_0\dot{\overline{\eta}} = \overline{q}_{i,i}. \qquad \text{----- (14)}$$

With these differences in mind, we can state and prove the uniqueness result. First, we consider the Biot's potential

$$U = \varrho(\varepsilon - T_o\eta) \text{ ---- (15)}$$

where ε is the internal energy per unit mass. Remember the fact that the free energy function ψ (the Helmholtz's function) is expressed as

$$\psi = \varepsilon - T\eta, \text{ ----- (16)}$$

By eliminating ε from (15) and (16), we deduce

$$\begin{aligned}
&U = \frac{1}{2}A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + B_{ijmn}\varepsilon_{ij}\gamma_{ij} + \frac{1}{2}C_{ijmn}\gamma_{ij}\gamma_{mn} +\\
&B_{ij}\sigma\varepsilon_{ij} + C_{ij}\sigma\gamma_{ij} + D_{ijk}\sigma_{,k}\varepsilon_{ij} + E_{ijk}\sigma_{,k}\gamma i_j + \qquad \text{----- (17)}\\
&d_i\sigma\sigma_{,i} - \frac{1}{2}a\theta^2 + \frac{1}{2}\xi\sigma^2 + \frac{1}{2}Ai_j\sigma_{,i}\sigma_{,j} - \frac{1}{2}\omega\dot{\sigma}^2.
\end{aligned}$$

Let K be the kinetic energy per unit mass, that is

$$K = \frac{1}{2}(\rho\dot{u}_i\dot{u}_j + I_{ij}\dot{\varphi}_i\dot{\varphi}_j + k\dot{\sigma}^2). \text{ ----- (18)}$$

In the next theorem we prove an estimation that will be used to prove the uniqueness result.

Theorem 1. *Let (u_i, φ_i, σ, θ) be a solution of the mixed initial-boundary value problem consists of the equations (11), the boundary conditions (10) and the initial conditions (9). Then the energy equation become:*

$$\begin{aligned}
&\frac{d}{dt}\int_B (U+K)dV = \int_B (\rho F_i\dot{u}_i + I_{ij}M_i\dot{\varphi}_j + \frac{\rho}{T_0}r\theta - \frac{1}{T_0}q_i\theta_i + f\dot{\sigma})dV + \qquad \text{--- (19)}\\
&\int_{\partial B} t_{ij}\dot{u}_j + m_{ij}\dot{\varphi}_j + h_i\dot{\sigma} + \frac{1}{T_0}q_i\theta)n_i dA.
\end{aligned}$$

Proof. With the aid of the constitutive equations (2) and the symmetry relations (7), we can write:

$$\begin{aligned}
&t_{ij}\dot{\varepsilon}_{ij} + m_{ij}\dot{\gamma}_{ij} + h_i\dot{\sigma}_{,i} - g\dot{\sigma} - \frac{\rho}{T_0}\mu\dot{\theta} =\\
&= \frac{1}{2}\frac{\partial}{\partial T}(A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + 2B_{ijmn}\varepsilon_{ij}\gamma_{mn} + C_{ijmn}\gamma_{ij}\gamma_{mn} + \qquad \text{--- (20)}\\
&2B_{ij}\sigma\varepsilon_{ij} + 2C_{ij}\sigma\gamma_{ij} + 2D_{ijk}\sigma_{,k}\varepsilon_{ij} + 2E_{ijk}\sigma_{,k}\gamma_{ij} -\\
&2\beta_{ij}\theta\varepsilon_{ij} - 2\alpha_{ij}\theta\gamma_{ij} - m\theta\sigma + 2d_i\sigma\sigma_{,i} + 2a_i\theta\sigma_{,i} -\\
&a\theta^2 + \xi\sigma^2 + A_{ij}\sigma_{,i}\sigma_{,j}) + \omega\dot{\sigma}.
\end{aligned}$$

On the other hand, in view of the equations of motions (4), equations of energy (6), the balance of the equilibrated forces (5) and the geometrical equations (3), it results:

$$\begin{aligned}
&t_{ij}\dot{\varepsilon}_{ij} + m_{ij}\dot{\gamma}_{ij} + h_i\dot{\sigma}_{,i} - g\dot{\sigma} - \frac{\rho}{T_0}\mu\dot{\theta} =\\
&(ti_j\dot{u}_j + m_{ij}\dot{\varphi}_j + h_i\dot{\sigma}_{,i} + \frac{1}{T}q_i\theta)_{,i} +\\
&\rho F_i\dot{u}_i + \rho M_i\dot{\varphi}_i + \rho L\dot{\sigma} + \frac{\rho}{T_0}r\theta - \frac{1}{T}q_i\theta_{,i} - \qquad \text{----- (21)}\\
&\frac{1}{2}\frac{\partial}{\partial t}(\rho\dot{u}_i\dot{u}_i + I_{ij}\dot{\varphi}_i\dot{\varphi}_j + \rho\kappa\dot{\sigma}^2) -\\
&\frac{\partial}{\partial t}(\beta_{ij}\varepsilon i_j\theta + \alpha_{ij}\gamma_{ij}\theta + m\sigma\theta + a_i\sigma_{,i}\theta).
\end{aligned}$$

By equalizing the right sides of the relations (20) and (21), we get:

$$\begin{aligned}&\frac{1}{2}\frac{\partial}{\partial t}(A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}+2B_{ijmn}\varepsilon_{ij}\gamma_{ij}+C_{ijmn}\gamma_{ij}\gamma_{mn}+2B_{ij}\sigma\varepsilon_{ij}+\\&2C_{ij}\sigma\gamma_{ij}+2D_{ijk}\sigma_{,k}\varepsilon_{ij}+2E_{ijk}\sigma_{,k}\gamma_{ij}-2\beta_{ij}\theta\varepsilon_{ij}-2\alpha_{ij}\theta\gamma_{ij}-\\&m\theta\sigma+2d_i\sigma\sigma_{,i}+2a_i\theta\sigma_{,i}+a\theta^2+\xi\sigma^2+A_{ij}\sigma_{,i}\sigma_{,j})+\\&\frac{1}{2}\frac{\partial}{\partial t}(\rho\dot{u}_i\dot{u}_i+I_{ij}\dot{u}_i\dot{u}_j+\rho\kappa\dot{\sigma}^2)=\\&\rho F_i\dot{u}_i+\rho M_i\dot{\varphi}_i+\rho L\dot{\sigma}+\frac{\rho}{T0}r\theta-\frac{1}{T0}q_i\theta_{,i}+\omega\dot{\sigma}^2+\\&(t_{ij}\dot{u}_j+m_{ij}\dot{\varphi}_j+h_i\dot{\sigma}+\frac{1}{T0}q_i\theta)_{,i}.\end{aligned} \quad \text{----- (22)}$$

Now, by integrating the identity (22) over B, we arrive at the desired result (19) and the proof of Theorem 1 is complete.

Based on the result of Theorem 1, we can prove the uniqueness of the solution for considered mixed problem.

Theorem 2. *If we assume that U is non-negative then there exist at most one solution for the problem given by the equations (11), the boundary conditions (10) and the initial conditions (9).*

Proof. Let $(u_i^{(1)}, \varphi_i^{(1)}, \sigma^{(1)}, \theta^{(1)})$ and $(u_i^{(2)}, \varphi_i^{(2)}, \sigma^{(2)}, \theta^{(2)})$ be two solutions of our problem corresponding to the same F_i, M_i, L and r and subject to same boundary conditions of the mixed type as in (10). We denote with $(\bar{u}_i, \bar{\varphi}_i, \bar{\sigma}_i, \bar{\theta})$ the difference of two solutions. For the difference, the relation (19) reads:

$$\frac{d}{dt}\int_B(\bar{U}+\bar{K})dV=\int_B\left(1\frac{1}{T_0}\bar{q}\bar{\theta}_{,i}-\omega\dot{\bar{\sigma}}^2\right)dV. \quad \text{----- (23)}$$

In view of (8), we obtain:

$$\frac{d}{dt}\int_B(\bar{U}+\bar{K})dV\le 0, \quad \text{---- (24)}$$

such that by integrating in (24) from 0 to t, we get:

$$\frac{d}{dt}\int_B(\bar{U}(0)+\bar{K}(0))dV\ge\frac{d}{dt}\int_B(\bar{U}(t)+\bar{K}(t))dV. \quad \text{----- (25)}$$

Because $(u_i^{(1)}, \varphi_i^{(1)}, \sigma^{(1)}, \theta^{(1)})$ and $(u_i^{(2)}, \varphi_i^{(2)}, \sigma^{(2)}, \theta^{(2)})$ satisfy the same initial dat, we conclude that the difference $(\bar{u}_i, \bar{\varphi}_i, \bar{\sigma}_i, \bar{\theta})$ corresponds to the null initial data, i.e.

$$\bar{u}_i=\bar{\varphi}_i=\bar{t}_{ij}=\bar{m}_{ij}=\bar{q}_i=0 \text{ on } \partial B\times[0,t_0],$$

such that (25) requires:

$$0\ge\frac{d}{dt}\int_B(\bar{U}(t)+\bar{K}(t))dV. \quad \text{----- (26)}$$

But $\bar{U}(t)$ *and* $\bar{K}(t)$ are positive denite and then they be zero everywhere in *B*. This means that the difference of solutions must vanish everywhere in *B* for all times and this complete the proof of Theorem 2.

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